Общи свойства на ядрата

Ядрен ъглов момент и четност.

Уравнение на Шродингер в централен потенциал

$$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t)=[-\frac{\hbar^2}{2m}\nabla^2+V(\vec{r})]\Psi(\vec{r},t)$$

$$[-\frac{\hbar^2}{2m}\nabla^2+V(\vec{r})]\Psi(\vec{r})=E\Psi(\vec{r})$$

$$\Psi(\vec{r},t)=\Psi(\vec{r})e^{-iEt/\hbar}$$

$$V(\vec{r})\equiv V(r,\theta,\varphi)=V(r)$$

Сферични координати:

{x,y,z} $\Longrightarrow$ {r.cos(φ)sin(θ), r.sin(φ)sin(θ), r.cos(θ)}

$$\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2 sin(\theta)}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2 sin^2(\theta)}\frac{\partial^2}{\partial\varphi^2}+\frac{2m}{\hbar^2}(V(r)-E)\right)\Psi=0$$

$$\Psi(r,\theta,\varphi)=Y(\theta,\varphi)R(r)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{dR}{dr}\right)+\frac{2m}{\hbar^2}(V(r)-E)R-AR=0$$

$$Y(\theta,\varphi)=\Theta(\theta)\Phi(\varphi)$$

$$\frac{1}{sin(\theta)}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right)+\frac{1}{sin^2(\theta)}\frac{\partial^2 Y}{\partial\varphi^2}+AY=0$$

$$\frac{1}{sin(\theta)}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)-\frac{B}{sin^2(\theta)}+A\Theta=0$$

$$\Theta_{lm}(\theta)=[\frac{2l+1}{2}\frac{(l-m)!}{(l+m)!}]^{1/2}P_l^m(cos(\theta))$$

$$\boxed{\begin{array}{l}A=l(l+1)\\ B=m^2\end{array}}$$

$$\frac{d^2\Phi}{d\varphi^2}+B\Phi=0$$

$$\Phi_m(\varphi)=\frac{1}{\sqrt{2\pi}}e^{im\varphi}$$

$$\boxed{Y_{lm}(\theta,\varphi)=(-1)^m\frac{e^{im\varphi}}{\sqrt{2\pi}}[\frac{2l+1}{2}\frac{(l-m)!}{(l+m)!}]^{1/2}P_l^m(cos\theta)\qquad l=0,1,2,\ldots\qquad m=0,\pm1,\pm2\ldots\pm l}$$

Централен потенциал – орбитален ъглов момент

$$-\frac{\hbar^2}{2m}\left(\frac{d^2R}{dr^2}+\frac{2}{r}\frac{dR}{dr}\right)+\left(\frac{l(l+1)\hbar^2}{2mr^2}+V(r)\right)R=ER \qquad \left\{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta})+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right\}Y=l(l+1)Y$$

$$V(\vec{r})\equiv V(r,\theta,\varphi)=V(r)$$

l^2 е интеграл на движението, т.е. $[H,l^2]=0$

$$\hat{\hat{L}}=-i\hbar\hat{\vec{r}}\times\nabla \quad -i\hbar\begin{vmatrix} i & j & k \\ x & y & z \\ \partial_x & \partial_y & \partial_z \end{vmatrix}$$

$$[l_x,l_y]=i\hbar l_z$$

$$l^2=l_x^2+l_y^2+l_z^2$$

$$[l^2,l_i]=0$$

$$l^2=(\vec{r}\times\vec{p}).(\vec{r}\times\vec{p})=r^2p^2+\hbar^2r^2\frac{\partial^2}{\partial r^2}+\hbar^2 2r\frac{\partial}{\partial r}$$

$$=\hbar^2\left\{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta})+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right\}$$

$$Y_{lm}(\theta,\varphi)=(-1)^m\frac{e^{im\varphi}}{\sqrt{2\pi}}\left[\frac{2l+1}{2}\frac{(l-m)!}{(l+m)!}\right]^{1/2}P_l^m(cos\theta)$$

$$l=0,1,2,\ldots \qquad m=0,\pm1,\pm2\ldots\pm l$$

$$\boxed{\begin{aligned}\hat{l}^2Y_{lm}(\theta,\varphi)&=\hbar^2l(l+1)Y_{lm}(\theta,\varphi)\\ \hat{l}_zY_{lm}(\theta,\varphi)&=\hbar mY_{lm}(\theta,\varphi)\end{aligned}}$$

$$Y_{00}=\frac{1}{\sqrt{4\pi}}$$

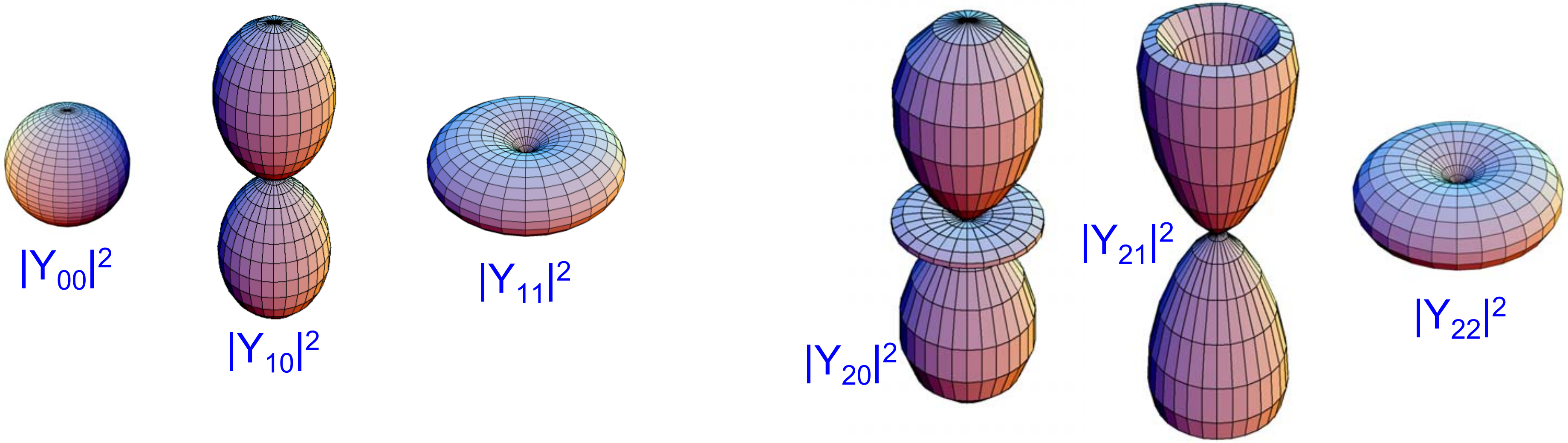
$$Y_{1-1}=\sqrt{\frac{3}{8\pi}}e^{-i\varphi}\sin(\theta) \qquad Y_{1-1}=\sqrt{\frac{3}{4\pi}}\cos(\theta) \qquad Y_{1-1}=\sqrt{\frac{3}{8\pi}}e^{+i\varphi}\sin(\theta)$$

$$Y_{2-1}=\sqrt{\frac{15}{8\pi}}e^{-i\varphi}\cos(\theta)\sin(\theta) \qquad Y_{20}=\sqrt{\frac{5}{16\pi}}(3\cos^2(\theta)-1) \qquad Y_{2+1}=\sqrt{\frac{15}{8\pi}}e^{i\varphi}\cos(\theta)\sin(\theta)$$

$$Y_{2-2}=\sqrt{\frac{15}{32\pi}}e^{-2i\varphi}\sin^2(\theta) \qquad Y_{2+2}=\sqrt{\frac{15}{32\pi}}e^{-2i\varphi}\sin^2(\theta)$$

За всяка стойност на орбиталното квантово число ***(l)*** съществуват ***2l+1*** решения свързани с магнитното квантово число (***m_l***)

Плътност, определена от ъгловата част



• За всеки централен потенциал големината на орбиталния ъглов момент е добро квантово число: $\langle l^2 \rangle = \hbar^2\, l(l+1),\ l=0, 1, 2, \ldots.$

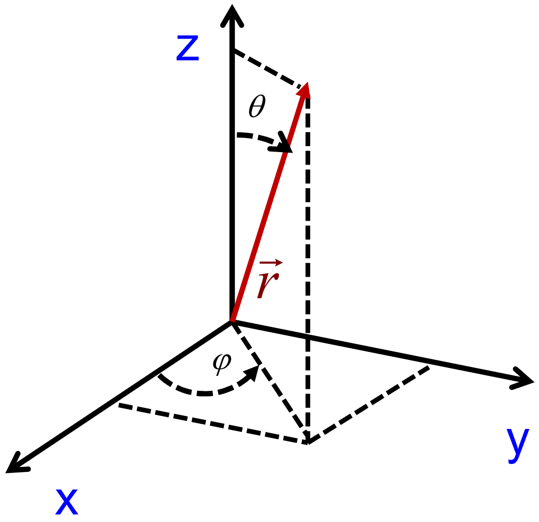
Спектрометрични означения

l	0	1	2	3	4	5	6
символ	s	p	d	f	g	h	i

• Възможно е да познаваме само една от компонентите на орбиталния ъглов момент

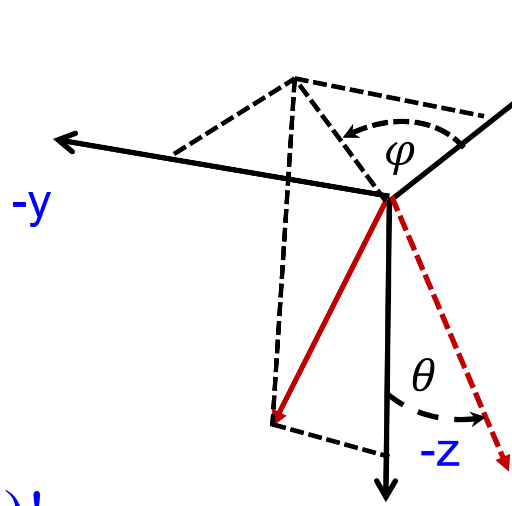
$$l_z = \hbar m_l,\ m_l=0, \pm1, \pm2, \ldots.\pm l$$

Орбитален ъглов момент и пространствено отражение



$$\Pi \equiv \begin{pmatrix} x \to -x \\ y \to -y \\ z \to -z \end{pmatrix}$$

$$\Pi \equiv \begin{pmatrix} r \to r \\ \theta \to \pi - \theta \\ \varphi \to \pi + \varphi \end{pmatrix}$$



$$Y_{lm}(\theta,\varphi) = (-1)^m \frac{e^{im\varphi}}{\sqrt{2\pi}}\left[\frac{2l+1}{2}\frac{(l-m)!}{(l+m)!}\right] P_l^m(cos\theta)$$

$$e^{im\varphi} \to e^{im(\varphi+\pi)} = (-1)^m e^{im\varphi}$$

$$\cos(\pi-\theta) = -\cos(\theta)$$

$$P_l^m(-z) = (-1)^{l-m} P_l^m(z)$$

$$\boxed{\Pi Y_{lm}(\theta,\varphi) = Y_{lm}(\pi-\theta,\pi+\varphi) = (-1)^l Y(\theta,\phi)}$$

$$\Pi\Psi(\vec{r}) = \Psi(-\vec{r}) = \pi\Psi(\vec{r})$$

$$\Pi^2\Psi(\vec{r}) = \pi^2\Psi(\vec{r}) = \Psi(\vec{r})$$

$$\pi = \pm$$

Орбитален ъглов момент

$$[l_x, l_y] = i\hbar l_z \qquad [l_+, l_-] = 2\hbar l_z \qquad \hat{l}^2 Y_{lm}(\theta,\varphi) = \hbar^2 l(l+1) Y_{lm}(\theta,\varphi)$$

$$[l^2, l_i] = 0 \qquad [l_z, l_\pm] = \pm\hbar l_\pm \qquad l_z Y_{lm}(\theta,\varphi) = \hbar m Y_{lm}(\theta,\varphi)$$

$$l = 0,1,2,.... \quad m = 0, \pm 1, \pm 2, ... \pm l$$

$$l_\pm = l_x \pm i l_y$$

$$[l_+, l_-] = (l_x + i l_y)(l_x - i l_y) - (l_x - i l_y)(l_x + i l_y)$$

$$= l_x^2 - i l_x l_y + i l_y l_x + l_y^2 - l_x^2 - i l_x l_y + i l_y l_x - l_y^2 = -2i[l_x, l_y] = -2i(i\hbar l_z) = 2\hbar l_z$$

$$l_+ l_z Y_{lm}(\theta,\varphi) = m\hbar l_+ Y_{lm}(\theta,\varphi) \qquad (l_z - \hbar) l_+ Y_{lm}(\theta,\varphi) = m\hbar l_+ Y_{lm}(\theta,\varphi)$$

$$l_+ l_z Y_{lm}(\theta,\varphi) = (l_z - \hbar) l_+ Y_{lm}(\theta,\varphi)$$

$$l_z(l_+ Y_{lm}(\theta,\varphi)) = (m+1)\hbar(l_+ Y_{lm}(\theta,\varphi))$$

$$l_z(l_- Y_{lm}(\theta,\varphi)) = (m-1)\hbar(l_- Y_{lm}(\theta,\varphi))$$

$$l_+ Y_{lm}(\theta,\varphi) = \alpha(l,m) Y_{l(m+1)}(\theta,\varphi)$$

$$l_- Y_{lm}(\theta,\varphi) = \beta(l,m) Y_{l(m-1)}(\theta,\varphi)$$

$$l_+ Y_{ll}(\theta,\varphi) = l_- Y_{l-l}(\theta,\varphi) = 0$$

повишаващ и понижаващ оператор

$$\int Y_{lm}^*(\theta,\varphi) l_- l_+ Y_{lm}(\theta,\varphi) d\Omega = |\alpha(l,m)|^2 \quad l^2 = l_x^2 + l_y^2 + l_z^2 = \frac{1}{2}(l_+ l_- + l_- l_+) + l_z^2 = \frac{1}{2}(2 l_- l_+ + 2\hbar l_z) + l_z^2 = l_- l_+ + l_z(l_z + \hbar)$$

$$\int Y_{lm}^*(\theta,\varphi)(l^2 - l_z(l_z + \hbar)) Y_{lm}(\theta,\varphi) d\Omega = \hbar^2[l(l+1) - m(m+1)] = |\alpha(l,m)|^2$$

$$l_+ Y_{lm}(\theta,\varphi) = \hbar\sqrt{l(l+1) - m(m+1)}\, Y_{l(m+1)}(\theta,\varphi)$$

$$l_- Y_{lm}(\theta,\varphi) = \hbar\sqrt{l(l+1) - m(m-1)}\, Y_{l(m-1)}(\theta,\varphi)$$

Алгебра на ъгловия момент

$$\hat{\vec{J}}:[J^2,J_i]=0$$

$$[J_x,J_y]=i\hbar J_z$$

$$\psi(j,m)$$

$$J^2\psi(j,m)=\hbar^2 j(j+1)\psi(j,m)$$

$$J_z\psi(j,m)=\hbar m\psi(j,m)$$

$$J_{\pm}\psi(j,m)=\hbar\sqrt{j(j+1)-m(m\pm1)}\psi(j,m\pm1)$$

всяко *j* задава (2*j*+1)-мерно пространство, в което неговите собствени функции образуват пълен базис

$$j=2\ (-2\le m\le 2)$$

$$\underset{\psi(j=2,m=2)}{\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}},\quad \underset{\psi(j=2,m=1)}{\begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix}},\quad ....,\underset{\psi(j=2,m=-2)}{\begin{bmatrix}0\\0\\0\\0\\1\end{bmatrix}}$$

$$J_+\psi(j,m)=a_{m,m+1}\psi(j,m+1)$$

$$J_+\Rightarrow\begin{pmatrix}0 & a_{1,2} & 0 & 0 & 0\\ 0 & 0 & a_{0,1} & 0 & 0\\ 0 & 0 & 0 & a_{-1,0} & 0\\ 0 & 0 & 0 & 0 & a_{-2,-1}\\ 0 & 0 & 0 & 0 & 0\end{pmatrix}$$

$$J_-\psi(j,m)=b_{m,m-1}\psi(j,m-1)$$

$$J_-\Rightarrow\begin{pmatrix}0 & 0 & 0 & 0 & 0\\ b_{2,1} & 0 & 0 & 0 & 0\\ 0 & b_{1,0} & 0 & 0 & 0\\ 0 & 0 & b_{0,-1} & 0 & 0\\ 0 & 0 & 0 & b_{-1,-2} & 0\end{pmatrix}$$

Вътрешен ъглов момент - спин

протони и неутроните имат вътрешен спин 1/2 – ъглов момент във спиновото пространство

$$\hat{\vec{S}} : [s^2, s_i] = 0 \qquad [s_+, s_-] = 2\hbar s_z$$

$$[s_x, s_y] = i\hbar s_z \qquad [s_z, s_\pm] = \pm\hbar s_\pm$$

$$\chi(s, s_z)$$

$$s^2\chi(s, s_z) = \frac{3}{4}\hbar^2\chi(s, s_z) \qquad s_\pm = s_x \pm i s_y$$

$$s_z\chi(s, s_z) = s_z\hbar\chi(s, s_z)$$

$$\hat{\vec{s}} = \frac{\hbar}{2}\hat{\vec{\sigma}} \qquad [\sigma_+, \sigma_-] = 4\sigma_z \qquad [\sigma_z, \sigma_\pm] = \pm 2\sigma_\pm$$

<u>2 – мерно пространство</u>

$$\chi(\frac{1}{2}, \frac{1}{2}) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \chi(\frac{1}{2}, -\frac{1}{2}) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\sigma_z\chi(1/2, 1/2) = \chi(1/2, 1/2)$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\sigma_z = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\sigma_z\chi(1/2, -1/2) = -\chi(1/2, 1/2)$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = -\begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$\boxed{\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}$$

Вътрешен ъглов момент - спин

$$\hat{\vec{S}} : [s^2, s_i] = 0$$

$$[s_x, s_y] = i\hbar s_z$$

$$[s_+, s_-] = 2\hbar s_z$$

$$[s_z, s_\pm] = \pm\hbar s_\pm$$

$$s_\pm = s_x \pm i s_y$$

$$\hat{\vec{s}} = \frac{\hbar}{2}\hat{\vec{\sigma}}$$

$$[\sigma_+, \sigma_-] = 4\sigma_z$$

$$[\sigma_z, \sigma_\pm] = \pm 2\sigma_\pm$$

$$\boxed{\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}$$

$$\sigma_+ = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} a & b \\ -c & -d \end{pmatrix} - \begin{pmatrix} a & -b \\ c & -d \end{pmatrix} = \begin{pmatrix} 0 & 2b \\ -2c & 0 \end{pmatrix} = \begin{pmatrix} 2a & 2b \\ 2c & 2d \end{pmatrix}$$

$$a = c = d = 0 \qquad b = 2$$

$$\sigma_+ = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} \qquad \sigma_- = \sigma_+^\dagger = \begin{pmatrix} 0 & 0 \\ 2 & 0 \end{pmatrix} \qquad \sigma_x^2 = \sigma_y^2 = \sigma_z^2 = \vec{1}$$

$$\sigma_x = \frac{1}{2}(\sigma_+ + \sigma_-) = \frac{1}{2}\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \frac{1}{2i}(\sigma_+ - \sigma_-) = \frac{1}{2i}\begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\hat{\vec{s}} = \frac{\hbar}{2}\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right)$$

матрици на Паули

Пълен ъглов момент на частица със спин 1/2

$$\psi(lm_l, 1/2m_s) = R_{nl}(r)Y_{lm_l}\chi_{1/2m_s}(\vec{\sigma})$$

$\{l^2, l_z, s^2, s_z\} - базис$

$$l^2\psi(lm_l, \frac{1}{2}m_s) = \hbar^2 l(l+1)\psi(lm_l, \frac{1}{2}m_s)$$

$$l_z\psi(lm_l, \frac{1}{2}m_s) = \hbar m_l \psi(lm_l, \frac{1}{2}m_s)$$

$$s^2\psi(lm_l, \frac{1}{2}m_s) = \hbar^2 \frac{3}{4}\psi(lm_l, \frac{1}{2}m_s)$$

$$s_z\psi(lm_l, \frac{1}{2}m_s) = \hbar m_s \psi(lm_l, \frac{1}{2}m_s)$$

$$[l^2, l_i] = 0$$

$$[l_x, l_y] = i\hbar l_z$$

$$[s^2, s_i] = 0$$

$$[s_x, s_y] = i\hbar s_z$$

Пълен ъглов момент

$$\boxed{\hat{\vec{J}} = \hat{\vec{l}} + \hat{\vec{s}} = \hat{\vec{l}} + \frac{\hbar}{2}\hat{\vec{\sigma}}}$$

$$[J_x, J_y] = i\hbar J_z$$

$$[J^2, J_i] = 0$$

$\{J^2, J_z, l^2, s^2\} - нов \ \ базис$

m_l и m_s не участват, т.к. $[J^2, \mathrm{l_z}] \neq 0, [J^2, \mathrm{s_z}] \neq 0$, но $[J^2, \mathrm{l_z} + \mathrm{s_z}] = 0$

$$J^2 = l^2 + s^2 + 2\vec{l}.\vec{s} = l^2 + s^2 + 2l_z s_z + 2l_x s_x + 2l_y s_y = l^2 + s^2 + 2l_z s_z + l_+ s_- + l_- s_+$$

$$l_+ s_- = (l_x + il_y)(s_x - is_y) = l_x s_x - il_x s_y + il_y s_x + l_y s_y$$

$$l_- s_+ = (l_x - il_y)(s_x + is_y) = l_x s_x + il_x s_y - il_y s_x + l_y s_y$$

Пълен ъглов момент на частица със спин 1/2

$$\psi(lm_l,1/2m_s)=R_{nl}(r)Y_{lm_l}\chi_{1/2m_s}(\vec{\sigma})$$

$\{J^2,J_z,l^2,s^2\}-нов\ \ базис$

m_l и m_s не участват, т.к. $[J^2,\mathrm{l_z}]\neq 0, [J^2,\mathrm{s_z}]\neq 0$, но $[J^2,\mathrm{l_z}+\mathrm{s_z}]=0$

$$J^2=l^2+s^2+2l_zs_z+l_+s_-+l_-s_+$$

$$[J^2,l_z]=[l_+,l_z]s_-+[l_-,l_z]s_+=-\hbar l_+s_-+\hbar l_-s_+$$

$$[J^2,s_z]=l_+[s_-,s_z]+l_-[s_+,s_z]=\hbar l_+s_--\hbar l_-s_+$$

$$[J^2,l_z+s_z]=0$$

$$l_+\psi(lm_l,\frac{1}{2}m_s)=\hbar\sqrt{l(l+1)-m_l(m_l+1)}\psi(lm_l+1,\frac{1}{2}m_s)$$

$$l_-\psi(lm_l,\frac{1}{2}m_s)=\hbar\sqrt{l(l+1)-m_l(m_l-1)}\psi(lm_l-1,\frac{1}{2}m_s)$$

$$s_+\psi(lm_l,\frac{1}{2}m_s)=\hbar\sqrt{s(s+1)-m_s(m_s+1)}\psi(lm_l,\frac{1}{2}m_s+1)$$

$$s_-\psi(lm_l,\frac{1}{2}m_s)=\hbar\sqrt{s(s+1)-m_s(m_s-1)}\psi(lm_l,\frac{1}{2}m_s-1)$$

$$J^2\psi(lm_l,\frac{1}{2}m_s)=\hbar^2\{l(l+1)+\frac{3}{4}+2m_lm_s\}\psi(lm_l,\frac{1}{2}m_s)+\alpha\psi(lm_l+1,\frac{1}{2}m_s-1)+\beta\psi(lm_l-1,\frac{1}{2}m_s+1)$$

Функциите $\psi(lm_l,1/2\,m_s)$ не са собствени на J^2. Кои са собствените му функции - $\boldsymbol{\psi(ls,jm)}$?

Коефициенти на Клебш-Гордън

Търсим преход от $\psi(lm_l, 1/2\, m_s)$ в $\boldsymbol{\psi(ls, jm)}$които са собствени на J^2

$$l\_\psi(lm_l, \frac{1}{2}m_s) = \hbar\sqrt{l(l+1) - m_l(m_l-1)}\psi(lm_l - 1, \frac{1}{2}m_s)$$

$$s\_\psi(lm_l, \frac{1}{2}m_s) = \hbar\sqrt{s(s+1) - m_s(m_s-1)}\psi(lm_l, \frac{1}{2}m_s - 1)$$

$$J\_ = l\_ + s\_$$

$$J\_\psi(ls, jm) = \hbar\sqrt{j(j+1) - m(m-1)}\psi(ls, jm-1)$$

$$\psi(ls, j = l + \frac{1}{2}m = l + \frac{1}{2}) = \psi(ll, \frac{1}{2}\frac{1}{2})$$

$$\psi(ls, j = l + \frac{1}{2}m = -l - \frac{1}{2}) = \psi(l - l, \frac{1}{2} - \frac{1}{2})$$

$$J\_\psi(ls, j = l + \frac{1}{2}m = l + \frac{1}{2}) = \hbar\sqrt{j(j+1) - j(j-1)}\psi(ls, j = l + \frac{1}{2}m = l + \frac{1}{2} - 1) = \hbar\sqrt{2j}\psi(ls, j = l + \frac{1}{2}m = l + \frac{1}{2} - 1)$$

$$= (l\_ + s\_)\psi(ll, \frac{1}{2}\frac{1}{2}) = \hbar\sqrt{2l}\psi(ll-1, \frac{1}{2}\frac{1}{2}) + \hbar\psi(ll, \frac{1}{2} - \frac{1}{2})$$

$$\psi(ls, j = l + \frac{1}{2}m = l + \frac{1}{2} - 1) = \left(\frac{2l}{2l+1}\right)^{1/2}\psi(ll-1, \frac{1}{2}\frac{1}{2}) + \left(\frac{1}{2l+1}\right)^{1/2}\psi(ll, \frac{1}{2} - \frac{1}{2})$$

$$|\psi(ls, j = l + \frac{1}{2}m = l + \frac{1}{2} - 1)|^2 = 1$$

$$m_l + m_s = l - \frac{1}{2}$$

$$m_l + m_s = l - \frac{1}{2}$$

Коефициенти на Клебш-Гордън

Търсим преход от $\psi(lm_l, 1/2\, m_s)$ в $\boldsymbol{\psi(ls, jm)}$ които са собствени на J^2

$$\psi(ls, j=l+\frac{1}{2} m=l-\frac{1}{2}) = \left(\frac{2l}{2l+1}\right)^{1/2} \psi(ll-1, \frac{1}{2}\frac{1}{2}) + \left(\frac{1}{2l+1}\right)^{1/2} \psi(ll, \frac{1}{2}-\frac{1}{2})$$

$$J_{\_}\psi(ls, j=l+\frac{1}{2} m=l+\frac{1}{2}-1) = \hbar\sqrt{2(2j-1)}\psi(ls, j=l+\frac{1}{2} m=l+\frac{1}{2}-2)$$

$$=(l_{\_}+s_{\_})\left(\left(\frac{2l}{2l+1}\right)^{1/2} \psi(ll-1, \frac{1}{2}\frac{1}{2}) + \left(\frac{1}{2l+1}\right)^{1/2} \psi(ll, \frac{1}{2}-\frac{1}{2})\right)$$

$$=\left(\frac{2l}{2l+1}\right)^{1/2} \hbar\sqrt{2(2l-1)}\psi(ll-2, \frac{1}{2}\frac{1}{2}) + \left(\frac{2l}{2l+1}\right)^{1/2} \hbar\psi(ll-1, \frac{1}{2}-\frac{1}{2}) + \left(\frac{1}{2l+1}\right)^{1/2} \hbar\sqrt{2l}\psi(ll-1, \frac{1}{2}-\frac{1}{2})$$

$$\psi(ls, j=l+\frac{1}{2} m=l+\frac{1}{2}-2) = \left(\frac{2l-1}{2l+1}\right)^{1/2} \psi(ll-2, \frac{1}{2}\frac{1}{2}) + \left(\frac{2}{2l+1}\right)^{1/2} \psi(ll-1, \frac{1}{2}-\frac{1}{2})$$

$$(J_{\_})^{j-m}\psi(ls, j=l+\frac{1}{2} m=l+\frac{1}{2}) = \left(l_{\_}+s_{\_}\right)^{j-m} \psi(ll, \frac{1}{2}\frac{1}{2})$$

$$\psi(ls, j=l+\frac{1}{2} m) = \left(\frac{l-m+1/2}{2l+1}\right)^{1/2} \psi(lm+\frac{1}{2}, \frac{1}{2}-\frac{1}{2}) + \left(\frac{l+m+1/2}{2l+1}\right)^{1/2} \psi(lm-\frac{1}{2}, \frac{1}{2}\frac{1}{2})$$

$$\psi(ls, j=l-\frac{1}{2} m) = -\left(\frac{l+m+1/2}{2l+1}\right)^{1/2} \psi(lm+\frac{1}{2}, \frac{1}{2}-\frac{1}{2}) + \left(\frac{l-m+1/2}{2l+1}\right)^{1/2} \psi(lm-\frac{1}{2}, \frac{1}{2}\frac{1}{2})$$

Пример – l=1

$$\psi(j=1+\frac{1}{2}, m=\frac{3}{2}) = \psi(l=1, m_l=1, \frac{1}{2}, \frac{1}{2})$$

$$J_-\psi(j=\frac{3}{2}, m=\frac{3}{2}) = \sqrt{\frac{3}{2}\frac{5}{2}-\frac{3}{2}\frac{1}{2}}\psi(j=\frac{3}{2}, m=\frac{1}{2}) = \sqrt{3}\psi(j=\frac{3}{2}, m=\frac{1}{2})$$

$$(l_-+s_-)\psi(l=1, m_l=1, \frac{1}{2}, \frac{1}{2}) = \sqrt{1.2-1.(1-1)}\psi(l=1, m_l=0, \frac{1}{2}, \frac{1}{2}) + 1.\psi(l=1, m_l=1, \frac{1}{2}, -\frac{1}{2})$$

$$\psi(j=\frac{3}{2}, m=\frac{1}{2}) = \sqrt{\frac{2}{3}}\psi(l=1, m_l=0, \frac{1}{2}, \frac{1}{2}) + \frac{1}{\sqrt{3}}\psi(l=1, m_l=1, \frac{1}{2}, -\frac{1}{2})$$

$$\psi(j=\frac{1}{2}, m=\frac{1}{2}) = \frac{-1}{\sqrt{3}}\psi(l=1, m_l=0, \frac{1}{2}, \frac{1}{2}) + \sqrt{\frac{2}{3}}\psi(l=1, m_l=1, \frac{1}{2}, -\frac{1}{2})$$

$$J_-\psi(j=\frac{3}{2}, m=\frac{1}{2}) = 2\psi(j=\frac{3}{2}, m=-\frac{1}{2})$$

$$(l_-+s_-)\left[\sqrt{\frac{2}{3}}\psi(l=1, m_l=0, \frac{1}{2}, \frac{1}{2}) + \frac{1}{\sqrt{3}}\psi(l=1, m_l=1, \frac{1}{2}, -\frac{1}{2})\right] =$$

$$= \sqrt{\frac{2}{3}}\sqrt{2}\psi(l=1, m_l=-1, \frac{1}{2}, \frac{1}{2}) + \frac{1}{\sqrt{3}}\sqrt{2}\psi(l=1, m_l=0, \frac{1}{2}, -\frac{1}{2}) + \sqrt{\frac{2}{3}}\psi(l=1, m_l=0, \frac{1}{2}, -\frac{1}{2})$$

$$\psi(j=\frac{3}{2}, m=-\frac{1}{2}) = \frac{1}{\sqrt{3}}\psi(l=1, m_l=-1, \frac{1}{2}, \frac{1}{2}) + \sqrt{\frac{2}{3}}\psi(l=1, m_l=0, \frac{1}{2}, -\frac{1}{2})$$

$$\psi(j=\frac{1}{2}, m=-\frac{1}{2}) = -\sqrt{\frac{2}{3}}\psi(l=1, m_l=-1, \frac{1}{2}, \frac{1}{2}) + \frac{1}{\sqrt{3}}\psi(l=1, m_l=0, \frac{1}{2}, -\frac{1}{2})$$

$$\psi(j=\frac{3}{2}, m=-\frac{3}{2}) = \psi(l=1, m_l=-1, \frac{1}{2}, -\frac{1}{2})$$

Коефициенти на Клебш-Гордън

$$\psi(ls, j=l+\frac{1}{2}m)=\left(\frac{l+m+1/2}{2l+1}\right)^{1/2}\psi(lm,\frac{1}{2}-\frac{1}{2})+\left(\frac{l-m+1/2}{2l+1}\right)^{1/2}\psi(lm,\frac{1}{2}\frac{1}{2})$$

$$\psi(ls, j=l-\frac{1}{2}m)=-\left(\frac{l-m+1/2}{2l+1}\right)^{1/2}\psi(lm,\frac{1}{2}-\frac{1}{2})+\left(\frac{l+m+1/2}{2l+1}\right)^{1/2}\psi(lm,\frac{1}{2}\frac{1}{2})$$

$$\{J_1^2, J_{1z}, J_2{}^2, J_{2z}\}$$

$$\psi(j_1,m_1,j_2,m_2)\equiv\psi(j_1,m_1)\psi(j_2,m_2)$$

$$\Longrightarrow$$

$$\vec{J}=\vec{J}_1+\vec{J}_2$$

$$\{J^2, J_z, J_1{}^2, J_2^2\}$$

$$\psi(j,m,j_1,j_2)=\sum_{\substack{m_1,m_2\\(m_1+m_2=m)}}\boxed{\langle j_1m_1, j_2m_2 \mid jm\rangle}\psi(j_1,m_1)\psi(j_2,m_2)$$

Ядрен ъглов момент и четност

Централен потенциал: $\vec{j} = \vec{l} + \vec{s}$ $\qquad \langle j^2 \rangle = \hbar^2 j(j+1)$

$\langle j_z \rangle = \langle l_z + s_z \rangle = \hbar m_j \qquad m_j = -j, -j+1, \ldots, j-1, j$

$$m_j = m_l + m_s = m_l \pm 1/2 \qquad m_j = \pm 1/2, \pm 3/2, \pm 5/2 \ldots \qquad j = \begin{cases} l + 1/2 \\ l - 1/2 \end{cases}$$

$$\vec{I} = \sum_{i=1}^{A} \vec{j}_i \qquad \langle I^2 \rangle = \hbar^2 I(I+1)$$

$$I_z = \hbar m = \hbar \sum m_j \qquad m = -I, -I+1, \ldots, I-1, I \qquad (2I+1)$$

j **е полуцяло число** ➡ ако A е четно → I_z е цяло → I е цяло

ако A е нечетно → I_z е полуцяло → I е полуцяло

$$\Pi \psi(\vec{r}) = \psi(-\vec{r}) = \pi \psi(\vec{r})$$

Централен потенциал: $\Pi \psi_j(\vec{r}) = (-1)^l \psi_j(-\vec{r})$

$$\pi = \pm \qquad \pi = \pi_1 . \pi_2 \ldots . \pi_A$$

Ядреният спин ***I*** и ядрената четност **π** са глобални характеристики на ядрото като цяло, т.е. трябва да се разглеждат (определят експериментално) като характеристики на “една” частица.